Shock-Free Turbomachinery Blade Design

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A computational method for designing shock-free, quasi-three-dimensional, transonic, turbomachinery blades is described. Shock-free designs are found by implementing Sobieczky's fictitious gas principle in the analysis of a baseline shape, resulting in an elliptic solution that is incorrect in the supersonic domain. Shock-free designs are obtained by combining the subsonic portion of this solution with a characteristic calculation of the correct supersonic flow using the sonic line data from the fictitious elliptic solution. This provides a new, shock-free blade design. Examples presented include the removal of shocks from two blades in quasi-three-dimensional flow and the development of a series of shock-free two-dimensional stators. The new designs all include modifications to the upper surface of an experimental stator blade developed at NASA Lewis Research Center. While the designs presented here are for inviscid flow, the same concepts have been successfully applied to the shock-free design of airfoils and three-dimensional wings with viscous effects. The extension of the present method to viscous flows is straightforward given a suitable analysis algorithm for the flow.

Introduction

WIDELY recognized requirements for lighter aircraft engine components with increased efficiency have forced the operating conditions of modern turbomachinery into the transonic regime. Once local regions of the internal flow become supersonic, however, it is only rarely that shocks do not occur.‡ Unfortunately, these shocks not only increase the inviscid drag through the production of entropy, they also adversely affect the boundary layer. Indeed, it has often been observed that if the Mach number just ahead of the shock foot exceeds 1.3, the boundary layer tends to separate, creating drag and the possibility of undesirable unsteady flow phenomena.

Sophisticated numerical methods recently have been developed which are capable of analyzing the complex internal flow in cascades.[1-5] Advances have also been made in the area of numerical design of shock-free cascades by Korn,[6] Garabedian and Korn,[7] and Dulikravich and Sobieczky.[8] The latter used the elliptic continuation principle, i.e., a fictitious gas, for designing shock-free transonic configurations. This paper reports the authors' success in applying this principle to the design of shock-free turbomachinery blades for transonic, two-dimensional and quasi-three-dimensional flows.

The fictitious gas method is simple in its formulation and limited only by the occurrence of limit lines in the otherwise shock-free flowfield. Limit lines, however, are not especially easy to circumvent. This requires experience in baseline redefinition and a knowledge of the operating conditions beyond which shock-free flows are not likely.

In two dimensions, an attempt to redesign a given stator blade to be shock-free was successful. It was necessary, however, to add some thickness to the upper surface of the baseline blade used in the fictitious gas calculation in order to obtain a design without a limit line. The results given for two-dimensional flow comprise a series of stator blades which were all developed from the same original stator by increasing the inlet Mach number. A unique feature of this process is that the original blade and the three new, higher Mach number designs can all be easily manufactured from a single casting.

The results for the quasi-three-dimensional case, while less extensive, are equally impressive. We note that by quasi-three-dimensional we mean that the flow is two-dimensional but with streamtube and radius variations included. The first case discussed is a highly loaded stator blade which originally resulted in a strong shock at design. The second case is a typical state-of-the-art rotor with streamtube and radius variations. Both blades were modified so that the new designs were shock-free at their respective operating conditions. For these cases it was not necessary to modify the thickness distribution of the baseline blades.

Due to the complexity of the internal flow problem, more than modest computational effort is necessary. The inviscid potential flow solution requires approximately 150 s for the two-dimensional case and 230 s for the quasi-three-dimensional case on a CDC 7600.

The practical consequences of this investigation should prove to be of interest to the aircraft engine industry. Its success will largely be dependent on the designers' ability to select good baseline configurations to reach the limiting operating conditions of shock-free flow.

Design Method

The design method is an application of Sobieczky's elliptic continuation principle to transonic turbomachinery flow. The method assumes that a reliable numerical analysis code exists. This code is modified so that the governing equations remain elliptic regardless of the flow type. This is accomplished most easily by altering the density-speed relationship in the hyperbolic domain.

The modified algorithm is used to determine the flowfield about a baseline profile. Being elliptic, this flowfield will contain no embedded shocks. However, it will be incorrect in the supersonic regime where a fictitious density has been used in the computations.

The correct supersonic flowfield is found using the method of characteristics. The calculations are performed in a hodograph-like plane, where the characteristics are orthogonal straight lines, by using the data on the sonic line of

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‡Shock-free flows are mathematically isolated from one another. Thus, any change in operating conditions will lead to a shock wave if the flow was originally shock-free.

the elliptic solution as initial data. The correct supersonic solution is transformed back to the physical plane where the $\psi = 0$ streamline defines the new body. The new profile will consist of the old profile in the region wetted by subsonic flow, and a modified profile in the supersonic region. The new profile will be tangent to, and have the same curvature as, the old profile at the sonic line-body intersection.

Formulation

As noted earlier, the procedure requires the modification of a numerical algorithm capable of describing the flowfield accurately. Farrell and Adamczyk's[3] artificial compressibility algorithm for the full potential equation was chosen for modification. This code calculates two-dimensional and quasi-three-dimensional flows. In the normal analysis mode this code solves

$$(\rho\phi_x)_x + (\rho\phi_y)_y = 0 \tag{1}$$

with

$$\rho = \rho_\infty \left\{ 1 + \frac{\gamma - 1}{2} M_\infty^2 \left[1 - \left(\frac{w^2 + \omega^2 (R_\infty^2 - R^2)}{w_\infty^2} \right) \right] \right\}^{1/(\gamma - 1)} \tag{2}$$

where w is the local relative velocity and for quasi-three-dimensional flow, R is the local radius and ω the angular speed of rotation.

Following the example of Sobieczky et al.,[9] we have chosen a fictitious gas for which the density is a function of the square of the local relative velocity. Thus, when hyperbolic behavior would normally be expected in solving Eq. (1), a gas law of the form

$$\rho = \rho_*(a_*/w)^P, \qquad P < 1 \ \text{for} \ w > a_* \tag{3}$$

replaces Eq. (2), thereby ensuring elliptic behavior. Therefore, the resultant flowfield is smooth. However, it is

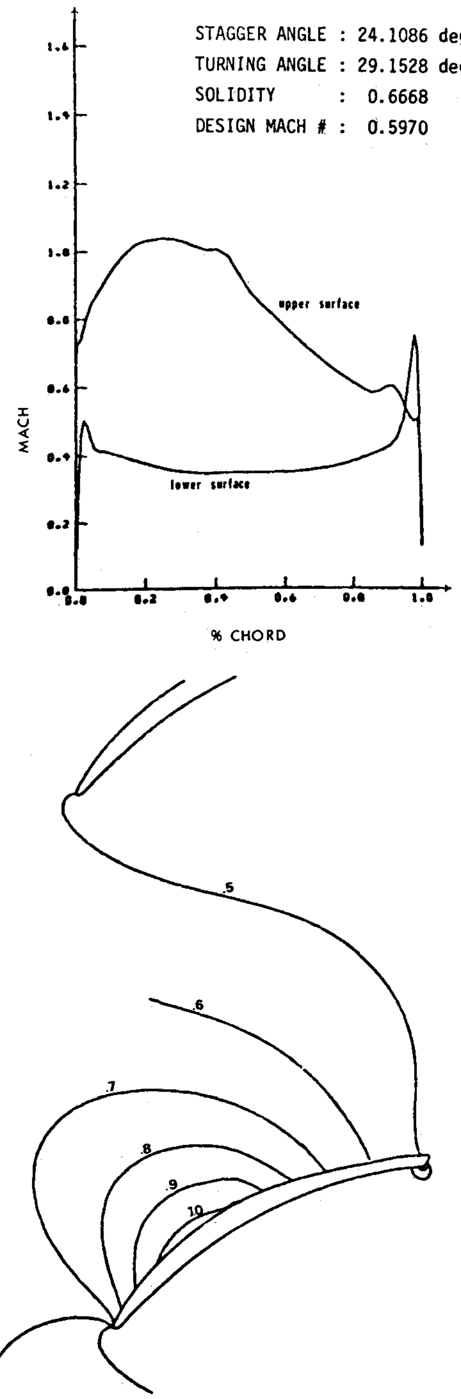

Fig. 1 Mach number distribution and isotachs for original blade (stator 7) of the design study. At this location the blade is just supercritical.

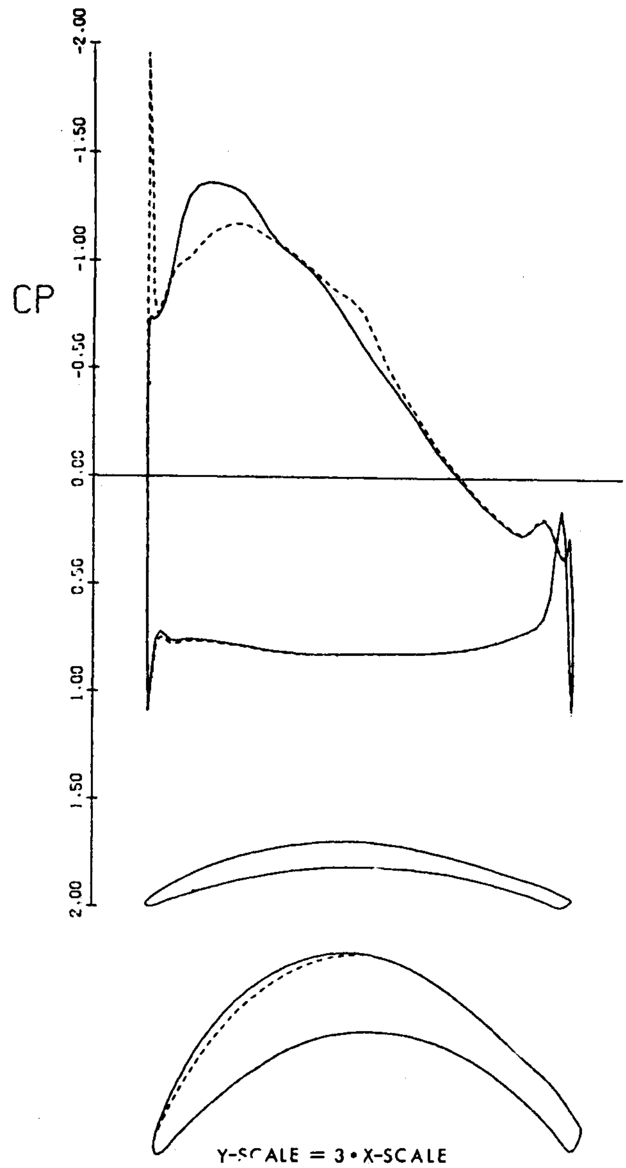

Fig. 2 Comparison of the two analyses for stator 6 at inlet Mach number $M = 0.625$ showing removal of leading-edge shock: (---), original blade and its pressure distribution; (——), redesigned blade and its pressure distribution. The original blade was thickened before applying the fictitious gas procedure.

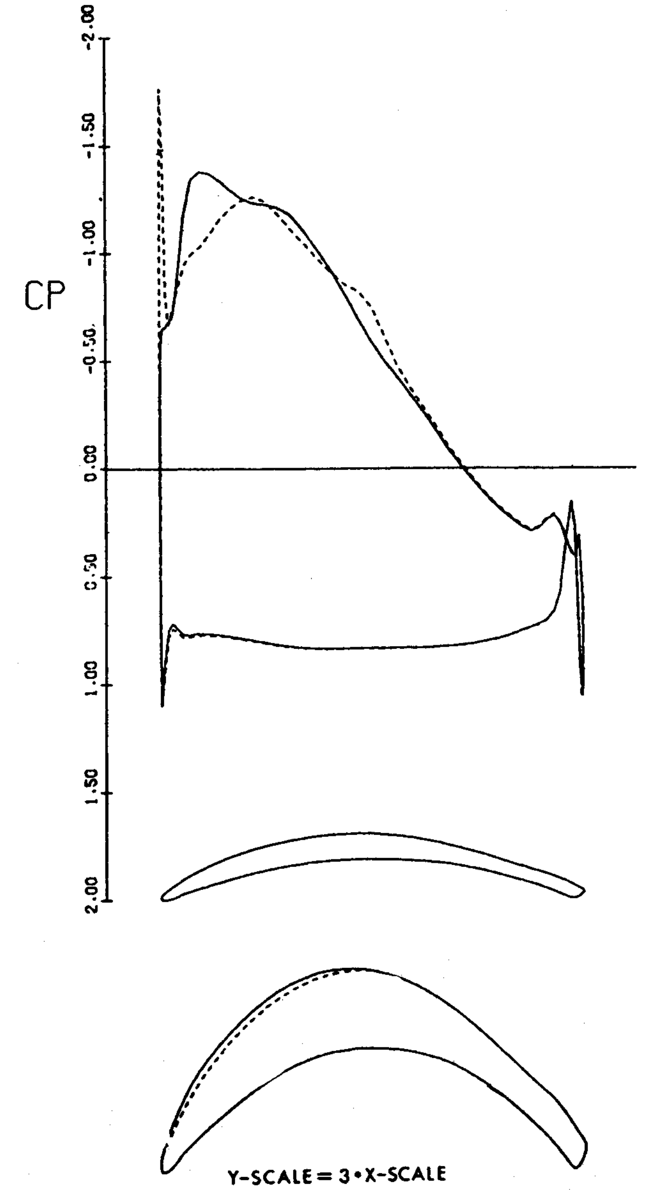

Fig. 3 Comparison of the two analyses for stator 5 at inlet Mach number $M = 0.660$ showing removal of leading-edge shock: (---), original blade; (——), redesigned blade. Both the correct vertical coordinates and three times these coordinates are used to display the blade shape.

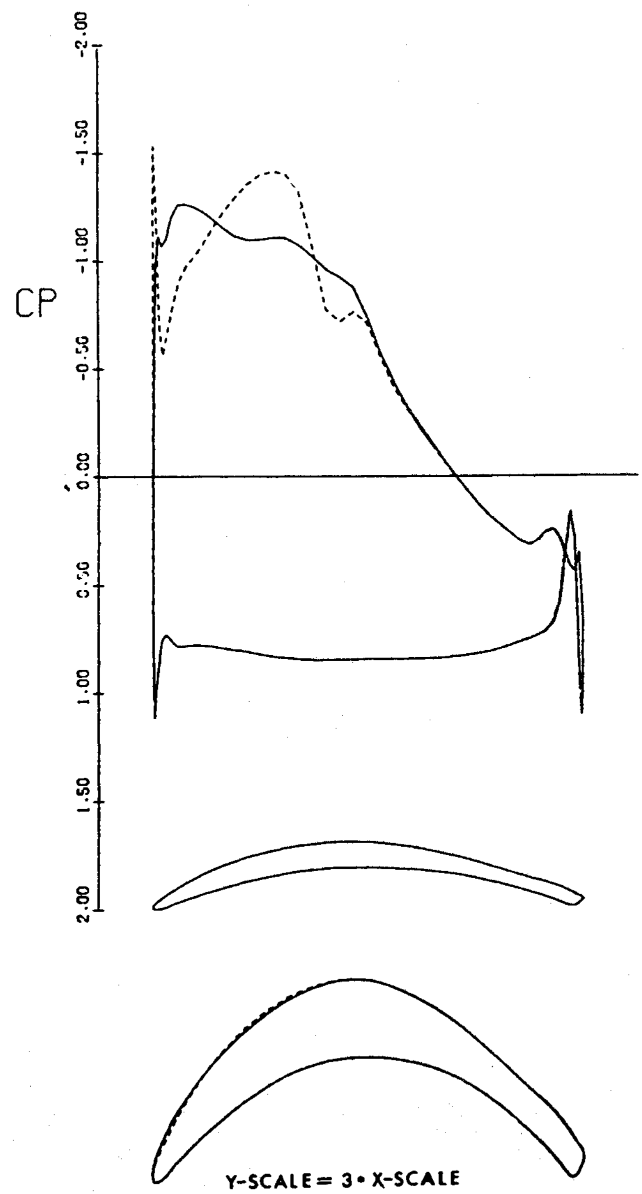

Fig. 4 Comparison of the two analyses for stator 4 at inlet Mach number $M = 0.700$ showing the removal of the leading-edge shock and a shock at 40% of chord: (---), original blade; (——), redesigned blade.

also incorrect in the supersonic domain, where the computations have been done using a density that is too large.

The calculation of the correct supersonic flowfield is carried out in hodograph-like variables using the method of characteristics. The transformation between the physical and hodograph planes is

$$dx + i dy = \frac{e^{i\theta}}{w}\left(d\phi + i\frac{\rho_0}{\rho b}d\psi\right) \tag{4}$$

or equivalently

$$d\phi = w[\cos(\theta)\,dx + \sin(\theta)\,dy]$$

$$d\psi = w\frac{\rho b}{\rho_0}[\cos(\theta)\,dy - \sin(\theta)\,dx] \tag{5}$$

These relations are much like the usual two-dimensional ones except for the inclusion of the ratio of local streamtube thickness to the upstream streamtube thickness, b.

From the elliptic solution, the spatial location of the sonic line and the corresponding flow deflection angles are determined. Equations (5) are then integrated to determine the values of ϕ and ψ on the sonic surface.

$$\phi = \int a_{*\infty}\frac{a_*}{a_{*\infty}}[\cos(\theta)\,dx + \sin(\theta)\,dy]$$

$$\psi = \int a_{*\infty}\frac{a_*}{a_{*\infty}}\frac{\rho_* b}{\rho_0}[\cos(\theta)\,dy - \sin(\theta)\,dx] \tag{6}$$

In the evaluation of Eqs. (6) it must be recognized that, unlike isolated airfoils, the value of $a_*/a_{*\infty}$ is not necessarily constant for turbomachinery. It may be shown from consideration of the rothalpy, which is constant on a streamline even in a rotating system with radius change, that

$$\frac{a_*^2}{a_{*\infty}^2} = 1 + \frac{\omega^2(R^2 - R_\infty^2)}{w_\infty^2}\left[1 + \frac{2}{M_\infty^2(\gamma - 1)}\right]^{-1} \tag{7}$$

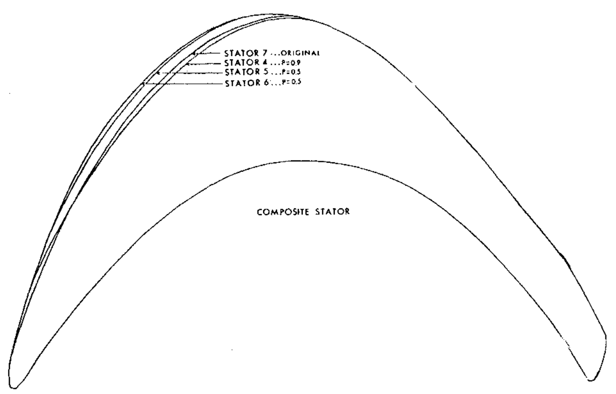

Fig. 5 Composite of the design study stators 4-7 depicting original blade shape and individual shapes of Figs. 1-4. *Y* scale used is three times that of *X* scale.

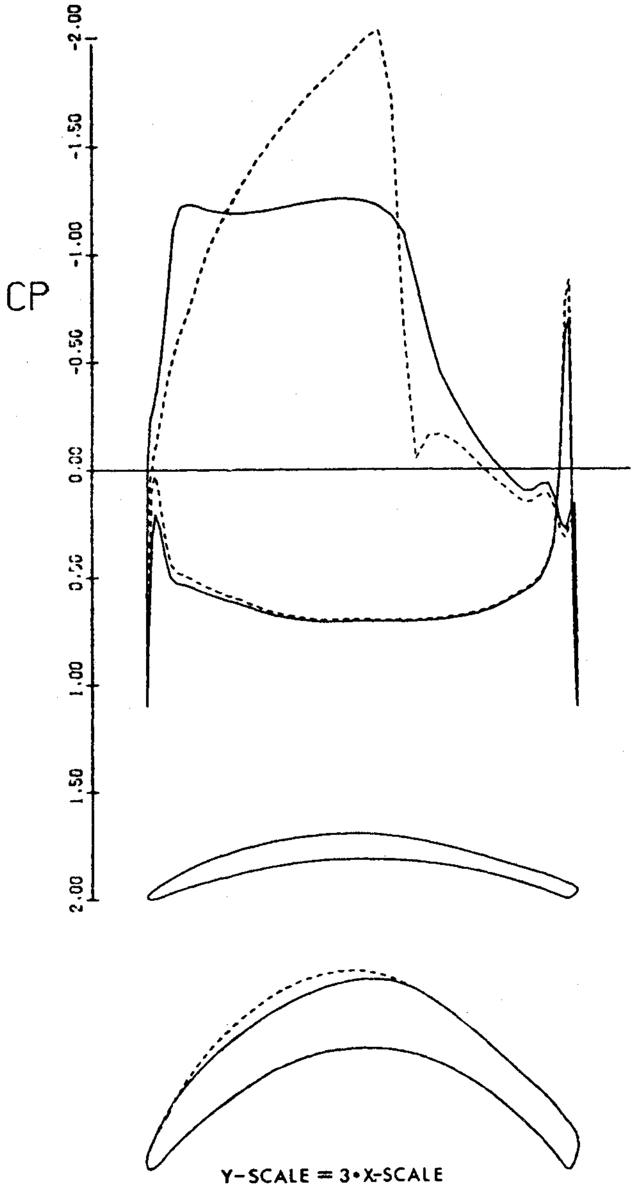

Fig. 6 Comparison of two analyses demonstrating removal of a strong shock from a quasi-three-dimensional stator at $M = 0.690$ by using the fictitious gas procedure.

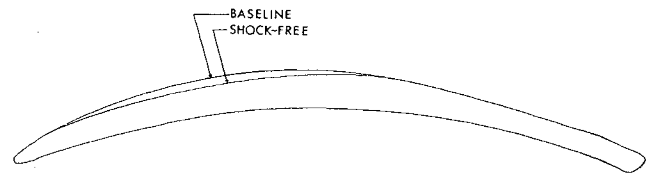

Fig. 7 Comparison of the blade shape before and after it has been redesigned to remove the strong shock in the quasi-three-dimensional case.

The integration of Eqs. (6) yields initial conditions which are then applied to determine the correct supersonic flowfield. If $\xi = \theta + \nu$ and $\eta = \theta - \nu$, where θ is the flow deflection angle and ν the Prandtl-Meyer turning angle, then the velocity potential and stream function satisfy

$$\phi_\xi = K(\nu)\psi_\xi, \qquad \phi_\eta = K(\nu)\psi_\eta \tag{8a}$$

where

$$K(\nu) = K[M(\nu)] = \frac{\rho_0}{\rho b}|M^2 - 1|^{1/2} \tag{8b}$$

Using the initial conditions, Eqs. (8) are solved using a method of characteristics with the usual density relationship. This determines the correct values of ϕ and ψ in the hyperbolic domain. The new airfoil shape is found by transforming the solution back to the physical plane using Eq. (4) and then determining the locus $\psi(x,y) = 0$.

A streamline in the hodograph plane may become tangent to a characteristic. When this occurs the transformation to the physical plane becomes multivalued and the solution must be discarded because it contains a limit line. It should be understood that limit lines are not a restriction on the method. Rather, they are an indication that no shock-free design exists for the given baseline at the chosen flow conditions with the fictitious gas used. The key to designing good blades is to know when a design can be obtained by altering the baseline geometry or the fictitious gas law and when no shock-free designs are possible.

Results

The possibility of redesigning a given stator blade to be shock-free at relatively high inlet Mach numbers has been explored using a two-dimensional blade shape developed by Schmidt[10] at NASA Lewis Research Center. The objective was to design a series of shock-free transonic stators which could be manufactured from a single blade shape. We consider a turbomachine in which the stages are numbered sequentially starting from the inlet, i.e., the first stage stator is at location 1. With stator location 7 representing the last supercritical stator, designs are obtained for the increasingly higher Mach numbers found by moving toward the inlet. The assumption is made that nothing changes from stator to stator except the inlet Mach number. No designs were found for stator locations 1, 2, and 3.

The results of this study are shown in Figs. 1-4. It is seen that increasing the inlet Mach number leads to the formation of a rapid expansion followed by a shock near the leading edge. At an inlet Mach number of 0.7 (stator 4), there is a second strong shock at 40% of chord. A baseline stator with strong leading-edge spikes tends to yield fictitious gas solutions which lead to limit lines. These were overcome for stators 4, 5, and 6 by adding thickness to the upper surface of the original blade. The process of adding thickness is not particularly difficult. For this case it involved three iterations requiring approximately 4 h (including computational time) for the three locations. The new designs, incorporating only minor surface changes, are all shock-free and were obtained using either the gas constant $P = 0.5$ or 0.9. That the objective

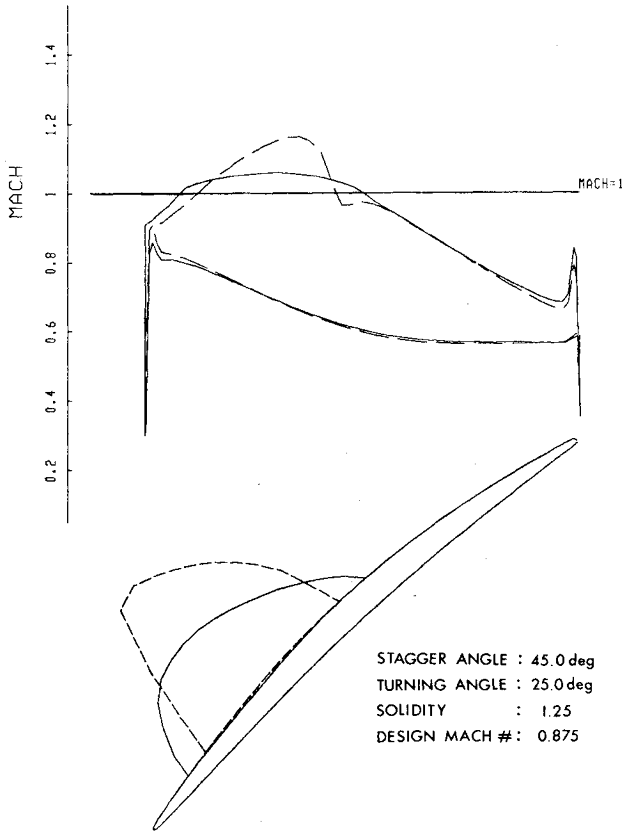

Fig. 8 Comparison of two analyses demonstrating the removal of a shock from a quasi-three-dimensional rotor at $M = 0.875$ by using the fictitious gas procedure: (---), original blade; (—), redesigned blade.

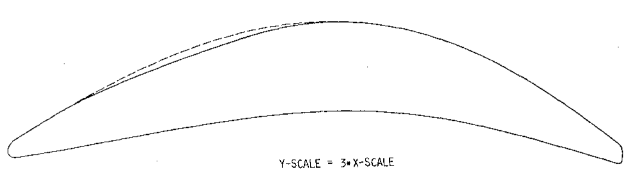

Fig. 9 Exploded view of the sonic zone showing the details of the blade shaving.

of making the blades all similar is met is demonstrated in Fig. 5 where the blade shapes are plotted as a composite.

The application of this method to quasi-three-dimensional cases is demonstrated here by the removal of a strong shock from a baseline blade provided by Schmidt.[10] Although geometrically similar to stator 7 of the previous study, this blade has an inlet design Mach number of 0.69 and a very strong shock at 60% of chord. Furthermore, the streamtube thickness ratio b of this blade is not constant. The comparison given in Figs. 6 and 7 clearly shows that the shock has been removed by removing some of the upper surface.

The final case to be shown is a rotor with radius change and streamtube variation included. Unlike the previous cases in which the large loading was created by low solidity and high turning, this case is more typical of state-of-the-art turbomachines. This case is also somewhat simpler to do, the more highly loaded cases being more likely to encounter limit lines. As shown in Figs. 8 and 9, a shock at 50% of chord has been eliminated by removing a portion of the upper surface.

Conclusion

The fictitious gas method has been applied successfully to the design of shock-free, transonic turbomachinery blades. A sequence of similar two-dimensional stator blades that are shock-free at their respective Mach numbers is provided. Shocks have been removed from both two-dimensional and quasi-three-dimensional cases. The only difficulty encountered—limit lines—was overcome for most flow conditions. Experience in using the method is required to judge when a shock-free design is no longer possible. In all cases, shock-free designs were obtained by modification of the thickness distribution of a baseline blade.

Acknowledgments

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